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JAN 79 D CALABRESE, M FISCHER, B HOIEM  
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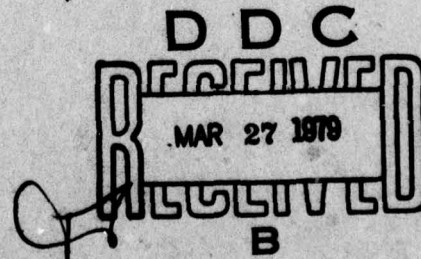


DEFENSE COMMUNICATIONS ENGINEERING CENTER

TECHNICAL NOTE NO. 6-79

AN ALGORITHM FOR PREDICTING THE  
PERFORMANCE OF A VOICE NETWORK  
WITH PREEMPTION

JANUARY 1979



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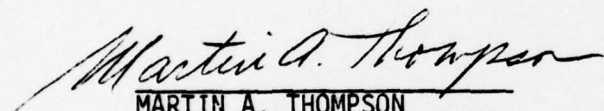
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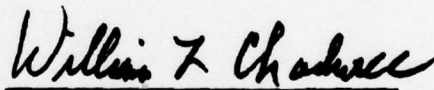
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FOREWORD

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## I. INTRODUCTION

✓ In order to evaluate the potential impact of facsimile-imagery service upon the current and future Automatic Voice Network (AUTOVON) and to reduce the time required to perform worldwide AUTOVON Flash Non-Blocking (FNB) Studies, the capability of the DCEC Voice Network Model has been extended from a single class traffic model to a two class traffic model. As in the AUTOVON system, one class of traffic can preempt the other class of traffic. Both ruthless and friendly preemption disciplines have been incorporated in the model for either originating office or spill control. This report documents the approach used by DCEC to extend the DCEC Voice Network Model to a two traffic class model with preemption.

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## II. BACKGROUND

DCEC's Voice Network Model (or algorithm) has been modified to include traffic capable of preemption. The details of the modification of this algorithm have been documented in this technical note. The algorithm was initially developed by Katz [1], implemented at DCEC by Fischer and Knepley [2], and incorporated into an overall voice network design and analysis algorithm (see Fischer, Garbin, Harris, and Knepley [3]). It is an analytical model based on queueing theory which converges to a steady state description of the network.

Since this algorithm provides the basis from which we start, it will be described briefly. For a more detailed discussion, see references [1], [2], and [3]. The algorithm assumes that the network topology is known. That is, the number and locations of the nodes of the network, the links and channel cross sections, point to point traffic requirements, and routing philosophy are all given. It also assumes:

a. Traffic originates at the nodes of the network as a Poisson process. For the preemptive network we assume that both classes of traffic originate in a Poisson process.

b. The length of time required to serve a call can be described by an exponential distribution. For the preemptive network we assume both classes of traffic have the same exponentially distributed holding time.

c. The network is in steady state.

d. The mean and variance of the offered load are sufficient to describe the behavior of a customer. This assumption is only used in the ruthless preemption discipline.

e. The behavior of calls on different links of the network is statistically independent.

f. Congestion in the network is only caused by the lack of available channels on the links.

In this network no queueing of calls is allowed and calls may search several paths in the network until they find a free one. In trying to mathematically describe what is happening, one is forced to consider a network of queues where at each queue no waiting is allowed and calls may have several paths to their destination from the same origin.

Some of the typical outputs that are available are shown in Figure 1. The same outputs are available in the modified algorithm for each class of traffic.

<u>OUTPUTS</u>	<u>NETWORK</u>	<u>NODE-TO-NODE</u>	<u>LINK</u>
GRADE OF SERVICE	X	X	
LINK UTILIZATION			X
LINK GRADE OF SERVICE			X
CONGESTION POINTS	X	X	

Figure 1. Typical Outputs of Voice Performance Algorithm

Typical numerical and computer time comparisons with the results of an event-by-event simulation are given in reference [3].

For the network with precedence traffic we assume there are two classes of calls trying to use the network, classes 1 and 2. Class 1 calls have priority over the class 2 calls. Both classes of calls use the same routing tables. Two types of preemption disciplines are considered. The first is known as ruthless; here, a class 1 call finding all the channels on a link busy preempts a class 2 call that occupies a channel. If no class 2 calls are present (i.e., class 1 calls are using all of the channels) the arriving class 1 call tries to find an alternate path in the network. Preempted class 2 calls are assumed lost. Class 2 calls arriving at a link try an alternate path if all channels in the link are busy.

The second type of preemption discipline that is considered is called friendly; here a class 1 call arriving at a link with all the channels busy tries to find an alternate path in the network to the destination. If all alternate paths are occupied, then the class 1 call returns to the original link and preempts a class 2 call that occupies a channel. Again if all the channels are occupied by class 1 calls, the call is considered blocked on that link and tries an alternate route. At the next link the same discipline is used. For the class 2 calls, the same philosophy and call disposition occurs as in the ruthless case.

The basic mathematical problem that one encounters is that the output process from a link is not Poisson and is not simply described. This is further complicated by the fact that there are three types of traffic trying to use any given link: originating, tandem, and overflow. Originating traffic is that traffic which has just entered the network; tandem traffic is that traffic carried previously by another link of the network; and overflow traffic is that traffic which has been blocked and is trying to find an alternate path to the destination. The arrival processes for each of these are different, and are not Poisson for the tandem and overflow (see Cooper [4]). Thus, each type of call sees a different blocking probability (see Kuczura [5]) and the evaluation of this probability is not straightforward (Kuczura [5] and Disney [6]). Thus, one is forced to consider some sort of procedure which approximates the behavior of the network. We have adapted an iterative procedure, first introduced by Katz [1], that uses the mean and variance of the offered load to describe the behavior of a call on a link.

The algorithm is an iterative procedure which is based on the assumption that the traffic flow in the network can be adequately described by the mean and variance of its distribution. This two-parameter characterization accounts for the non-Poisson nature of the traffic which overflows or is carried by a link.

At each iteration, a two-step procedure is implemented in the algorithm. The first step is a load assignment procedure. During this step the offered traffic is distributed over the network (one source-destination pair at a time) using the routing table and the link blocking probabilities calculated at the last iteration. Initially, these probabilities are set to zero. By considering all source-destination pairs of the network, the offered load to each link of the network and the total traffic lost for all pairs are accumulated during the load assignment procedure.

The next step in the current iteration is to compute new estimates of the link blocking probabilities and the mean and variance of the overflow processes. From the load assignment procedure we know the mean and variance of the offered load to each link of the network. Based on the relationship between these parameters, three methods are used to determine blocking probabilities and moments of overflow processes. If the mean and variance are equal, we then assume the offered load is being generated by a Poisson process and Erlang's Loss Formula is used to give an expression for the blocking probability. For the case where the mean is less than the variance, the process is basically one for which the traffic has overflowed another link (see Cooper [4]) and Wilkinson's Equivalent Random Technique (see [4]) is used to obtain the desired results. For the case where the mean is greater than the variance, the Smoother Than Random Technique (see Katz [1]) is used.



If the new estimates are not sufficiently close to the computed values of the previous iteration, the load assignment procedure is repeated using the new values of link blocking. The algorithm converges when the link blocking probabilities computed from a set of offered loads are the same as those used to generate those loads. Typically, convergence requires from 7 to 10 iterations.

We have modified the basic voice performance algorithm to handle either of the preemption disciplines mentioned above. The current version of the algorithm assumes the same discipline is used throughout the network. In this technical note the philosophy of the required modifications is discussed (section III). In section IV some numerical and time comparisons with the results of an event-by-event simulation are given. The conclusions and future work are discussed in section V. For completeness, an appendix with three subsections is included. In the first subsection of the appendix the mathematical analysis of the link performance of the friendly discipline is given. Subsection 2 gives a mathematical treatment of a link reliability/performance analysis required in the Flash Non Blocking Study. The last subsection contains a detailed discussion of the load reduction procedure used in the basic network design algorithm.

### III. MODIFICATIONS TO THE BASIC ALGORITHM

From section II, one can see that a vital ingredient of the basic performance algorithm is the capability to predict link performance. Thus, for each of the preemption disciplines, ruthless and friendly, one must be able to predict the link performance for each class of traffic. Once the required equations have been developed, the appropriate changes to the basic algorithm have to be made. In this section, we discuss the link models and the subsequent changes required.

First, let us consider the link performance of the ruthless preemption discipline. We assume that the arrival process of each class to the link is an independent Poisson process, and that the link has  $s$  channels. The load for class  $i$  calls is  $\rho_i$  erlangs ( $i=1,2$ ). As in section II, the service holding times are exponentially distributed with the same mean for each class of calls. Class 1 calls have precedence over class 2 calls in the sense that if all the channels are busy when a class 1 call arrives, the class 1 call will preempt a class 2 call on the link. If there are only class 1 calls on the link, the arriving class 1 call is alternately routed. The preempted class 2 call is assumed lost and is not alternately routed. Only arriving class 2 calls which find all channels busy are alternately routed. We are interested in determining the steady state loss probability for each class, denoted by  $PL_i, i=1,2$ . It should be pointed out that one minus this measure of performance is the proportion of class  $i$  traffic that completes its call on this link. The quantity  $PL_i$  does not always represent the blocking probability for class  $i$  on the link. The blocking probability is the probability an arriving call finds all the channels busy.

The mathematical analysis of this preemption discipline has been considered by Burke [7]. If  $E(a,s)$  is Erlang's loss formula, that is

$$E(a,s) = \frac{a^s/s!}{\sum_{n=0}^s a^n/n!},$$

then

$$PL_1 = E(\rho_1, s), \quad (1)$$

and

$$PL_2 = E(\rho_1 + \rho_2, s) + \frac{\rho_1 \{E(\rho_1 + \rho_2, s) - E(\rho_1, s)\}}{\rho_2} \quad (2)$$

Equation (1) is easy to see, since the class 1 calls preempt class 2 calls when blocked; the only traffic class 1 calls have to contend with are from its own class. Erlang's Loss Formula,  $E(a, s)$ , represents the probability of  $s$  busy channels with an offered load of  $a$  erlangs. The only time a class 1 call does not receive service from the link (and hence is lost) is when all the channels are busy with class 1 calls. Equation (1) then follows.

Equation (2) requires a little more explanation. Class 2 calls are denied service from the link in two ways. First, all the channels may be busy on the initial attempt, and, second, a class 2 call may be preempted after it has started to use one of the channels. It can be shown that the probability that all  $s$  channels are busy, regardless of the class of calls using the link, is given by Erlang's Loss Formula with the offered load equal to the sum of the class 1 and class 2 loads; i.e.,  $a = \rho_1 + \rho_2$ . This factor is given by the first term in the right-hand side of equation (2).

The second way a class 2 call does not receive service from the link is by preemption. This factor is given by the second term in the right-hand side of equation (2); but let us look at it more closely. The quantity  $E(\rho_1 + \rho_2, s) - E(\rho_1, s)$  represents the probability that class 1 calls preempt class 2 calls; this factor multiplied by the class 1 load,  $\rho_1$ , gives the class 2 load that is preempted. The whole quantity divided by the offered class 2 load,  $\rho_2$ , is the portion of class 2 calls that are preempted on the link. This quantity is the second term in the right-hand side of equation (2).

From earlier discussions on the effect of a ruthless preemption discipline on class 1 calls, one can see that there is no change in the basic algorithm to determine the network performance for the class 1 calls. In fact, one may use the two-moment version of the algorithm to describe the behavior for class 1 calls. For all the other cases that we consider, only a one-moment characterization of the offered traffic is used.

For class 2 calls changes are required. First, equation (2) is used at the end of the iteration to update the link measures of performance, based on the current estimate of the offered class 1 and 2 loads. During the load assignment procedure for class 2 traffic, there are some additional changes to be made, pictorially represented in Figure 2.



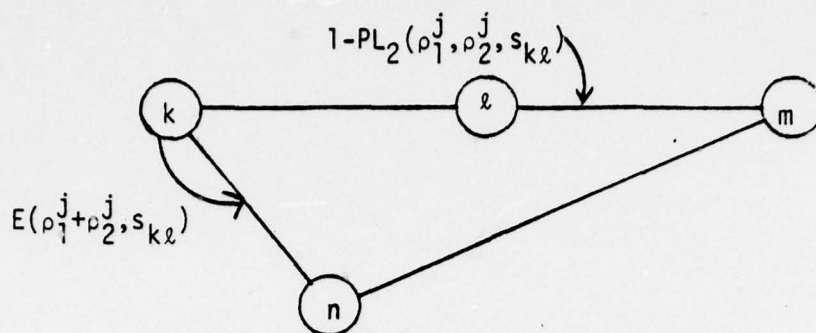


Figure 2. Load Assignment Procedure for Class 2 Calls

Let  $\rho_i^j, i=1,2$ , be the estimated class  $i$  load on link  $(k,l)$  from the  $j^{\text{th}}$  iteration, and  $s_{k,l}$  the number of channels on link  $(k,l)$ . In the  $(j+1)^{\text{st}}$  iteration, suppose a class 2 call wants to go from node  $k$  to node  $m$ , and the routing dictates that blocked calls on link  $(k,l)$  overflow to link  $(k,n)$ . Class 2 calls would overflow to link  $(k,n)$  with probability  $E(\rho_1^j + \rho_2^j, s_{kl})$ , while class 2 calls would be carried by link  $(k,l)$  with probability  $1-PL_2(\rho_1^j, \rho_2^j, s_{kl})$ , where  $PL_2(\rho_1^j, \rho_2^j, s_{kl})$  is equation (2) with parameters  $\rho_1^j, \rho_2^j$ , and  $s_{kl}$ . That is, class 2 calls are overflowed via the blocking (all channels busy) formula and tandemed with the probability it is carried by link  $(k,l)$ . We note from equation (2) that  $PL_2(\rho_1^j, \rho_2^j, s_{kl})$  is greater than or equal to  $E(\rho_1^j + \rho_2^j, s_{kl})$ .

For the friendly preemption discipline, more problems arise. Since a blocked class 1 call tries to find another path in the network before searching the link in a ruthless manner, the link performance analysis is difficult. In order to make the mathematical analysis tractable, we formulate the problem as shown in Figure 3.

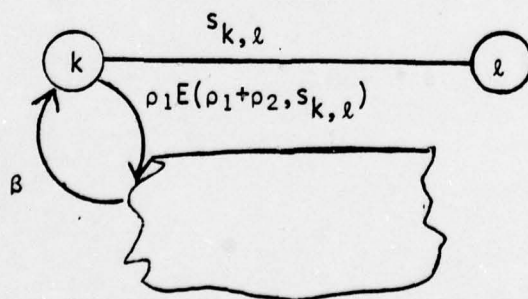


Figure 3. Link System Formulation for Friendly Search

Let  $\rho_i, i=1,2$  be the offered class  $i$  load to a link  $(k,l)$  with  $s_{k,l}$  channels, and the statistical independence assumptions be the same as in the ruthless case. An arriving class 1 call that finds all the channels busy tries an alternate path in the network. If all the paths are occupied, it returns and searches link  $(k,l)$  in a ruthless manner. We assume that the probability of returning is  $\beta$ . The complete mathematical analysis of this system is given in the appendix.

We are interested in determining the loss probability,  $PL_1$ , for each class of calls. If  $Q_1$  represents the number of class 1 calls on the link in steady state, then

$$PL_1 = (1-\beta)E(\rho_1+\rho_2, s_{k,l}) + \beta \Pr\{Q_1 = s_{k,l}\} \quad (3)$$

and

$$PL_2 = E(\rho_1+\rho_2, s_{k,l}) + \frac{\rho_1 \beta \{E(\rho_1+\rho_2, s_{k,l}) - \Pr\{Q_1 = s_{k,l}\}\}}{\rho_2} \quad (4)$$

Let us first consider the loss probability for the class 1 calls,  $PL_1$ . The quantity  $E(\rho_1+\rho_2, s_{k,l})$  represents the probability all the channels are busy and hence the probability an arriving class 1 call tries to find an alternate path in the network. Since  $1-\beta$  is the probability the call does not return to the link, the first part on the right-hand side of equation (3) is the portion of arriving class 1 calls that find all the channels busy and a free alternate path. These calls are considered lost to link  $(k,l)$ .

There is another way calls can be lost to the link. The probability  $\Pr\{Q_1 = s_{k,l}\}$  is the probability the channels are all occupied by class 1 calls. If we multiply this quantity by the probability all the alternate paths are occupied,  $\beta$ , the product represents the portion of class 1 calls that find all the channels on the link occupied by class 1 calls and all the alternate paths busy. The quantity is given by the second part in equation (3).

The development of the loss probability,  $PL_2$ , for class 2 calls is similar to the case of ruthless preemption. The only quantity that needs some explanation is the numerator of the second factor in equation (4). The quantity  $E(\rho_1+\rho_2, s_{k,l}) - \Pr\{Q_1 = s_{k,l}\}$  is the probability that class 1 calls preempt class 2 calls. This quantity multiplied by the amount of Class 1 traffic capable of preempting class 2 traffic,  $\beta\rho_1$ , is the amount of preempted class 2 traffic. Dividing this quantity by the class 2 traffic offered to the link gives the probability that a class 2 call will be preempted. The quantities  $\rho_1$  and  $\rho_2$  are used as they were in the ruthless case.

Equations (3) and (4) are used in the algorithm in exactly the same way as equations (1) and (2) were in the ruthless preemption discipline. So changes to the basic algorithm follow along the same line as

discussed for the ruthless discipline. There still remain two problems before equations (3) and (4) can be implemented. We have not given an easily computable expression for  $\Pr\{Q_1=s_{k,2}\}$ . How does one determine  $\beta$  in the algorithm? As we mentioned earlier, a complete mathematical analysis of the link performance is given in the appendix. From that, one can determine  $\Pr\{Q_1=s\}$  (note that we have dropped the subscripts indicating the link under consideration) from the solution of  $s-1$  linear equations. This type of solution would not be feasible for implementation in our algorithm because it would require too much computer run time.

One is then forced to consider some sort of approximation for  $\Pr\{Q_1=s\}$ . The details of the approximation are given in the appendix; but the basics are as follows. We have examined some numerical data where we have numerically solved for the correct  $\Pr\{Q_1=s\}$ . This investigation indicated that  $\Pr\{Q_1=s\}$  had a quadratic form in  $\beta$ . Let us denote the approximation to  $\Pr\{Q_1=s\}$  by  $f(\rho_1, \rho_2, s, \beta)$  and thus

$$f(\rho_1, \rho_2, s, \beta) = C_0 + C_1\beta + C_2\beta^2, \quad (5)$$

where the constant  $C_0$ ,  $C_1$  and  $C_2$  have to be determined.

When  $\beta=0$  the blocked class 1 calls do not return and the behavior on the link is as if both classes of calls were equally contending for the channels. From Cooper [4], one can show

$$\Pr\{Q_1=s\} = \frac{\rho_1^s / s!}{\sum_{n=0}^s (\rho_1 + \rho_2)^n / n!}. \quad (6)$$

Similarly, if  $\beta=1$  then the class 1 calls appears as if they were operating in the ruthless mode and

$$\Pr\{Q_1=s\} = E(\rho_1, s). \quad (7)$$

Equations (6) and (7) provide two of three equations necessary for determining the three unknowns ( $C_0$ ,  $C_1$ ,  $C_2$ ); the other equation is found by examining numerical data and relating the parameters to the boundary condition  $\beta=1$ :

$$(\rho_1 + \rho_2)f(\rho_1, \rho_2, s, 0.6) \approx \rho_1 f(\rho_1, \rho_2, s, 1) \quad (8)$$

where  $\approx$  means approximate.



Equations (6), (7), and (8) provide three equations in  $C_0$ ,  $C_1$ , and  $C_2$ . The solution of these equations and the comparison of the approximation to the exact solution are given in the appendix.

The other problem is the determination of  $\beta$  for each link of the network. Remembering that  $\beta$  represents the probability of an arriving class 1 call returning to the link after searching all the alternate paths in a friendly mode, we essentially set  $\beta$  for each link to

$$\beta = \prod_{k \in A} E(\rho_1(k) + \rho_2(k), s_k) , \quad (9)$$

where  $A$  is the set of alternate links for the link under consideration,  $\rho_i(k)$  the class  $i$  load on the  $k$ th link, and  $s_k$  the number of channels in the link. In reality,  $\beta$  should be directionalized; that is, calls going from node  $k$  to node  $\ell$  have a different  $\beta$  than calls from node  $\ell$  to  $k$ . This fact has not been incorporated into the current version of the algorithm. At the end of each iteration a new  $\beta$  is computed for each link.

In the next section, the current version of the voice preemption model is compared with the results of an event by-event simulation.

#### IV. NETWORK MODEL PERFORMANCE

The performance of the DCEC Voice Network Model with Preemption was determined for both the ruthless and friendly preemption disciplines. For each preemption discipline, five event-by-event computer simulation runs (each using a different random seed) were performed for three different network configurations representative of the Pacific, CONUS, and European AUTOVON. Next, using the traffic data from the simulation runs as input to the DCEC Voice Network Model with Preemption, the network and switch-to-switch grades of service (GOS's) were estimated by the model for each configuration. The 95% confidence intervals for the network and switch-to-switch GOS's obtained from the computer simulation runs were computed using Student's t distribution. The GOS's estimated by the model were then compared to the average GOS's for the five computer simulation runs and the number of estimates from the model which fell within the confidence interval for the computer simulation runs was determined.

Figure 4 shows the results of the analysis of the performance of the DCEC Voice Network Model with Preemption for the ruthless preemption discipline. For each of the three configurations over three fourths of the switch-to-switch GOS's fell within the 95% confidence interval derived from the results of the computer simulation runs of the performance of each network configuration.

Since the network GOS for first class traffic was less than or equal to 0.05 for the CONUS and European configurations, a dual moment model for class 1 traffic was used in order to maintain the accuracy of the model. (Below a network GOS of 0.05 a dual moment model is usually necessary in order to maintain accuracy.)

Figure 5 shows the results of the analysis of the performance of the DCEC Voice Network Model with Preemption for the friendly preemption discipline. With the exception of the CONUS configuration, over three fifths of the switch-to-switch GOS's fell within the 95% confidence interval derived from the results of the computer simulation runs of the performance of each configuration. For the Pacific network configuration, dual moment modeling was not necessary since the network GOS for each class of precedence traffic exceeded 0.05. Using a single moment model for CONUS, only one third of the number of switch-to-switch GOS's fell within the 95% confidence interval for class 1 traffic (the overall network GOS equaled .04). No dual moment model as yet has been implemented for the friendly preemption discipline. Although the European network also experienced a network GOS less than 0.05 for class 1 traffic the percentage of switch-to-switch pairs whose GOS fell within the 95%

# RUTHLESS PREEMPTION

NETWORK CONFIGURATION	NETWORK		SOURCE SW TO DESTINATION SW GOS		COMMENTS
	GRADE OF SERVICE (GOS) EVENT-BY-EVENT SIMULATOR (5 RUNS)	ANALYTICAL MODEL	MEAN ABSOLUTE ERROR	WITHIN 95% CONFIDENCE INTERVAL	
PACIFIC (6 SW)	.173 (Class 1)	.177	.006	93%	Single Moment Model Spill Control 209 erlangs
	.593 (Class 2)	.597	.018	76%	
CONUS (10 SW)	.036 (Class 1)	.032	.008	76%	Dual Moment Model Spill Control
	.659 (Class 2)	.644	.025	87%	
EUROPE (11 SW)	.051 (Class 1)	.048	.006	85%	Dual Moment Model Originating Office Control
	.519 (Class 2)	.514	.024	79%	

Figure 4. Performance DCEC Voice Network Model with Preemption - Ruthless Discipline



# FRIENDLY PREEMPTION

NETWORK CONFIGURATION	NETWORK		SOURCE SW TO DESTINATION SW GOS		COMMENTS
	GRADE OF SERVICE (GOS) EVENT-BY-EVENT SIMULATOR (5 RUNS)	ANALYTICAL MODEL	MEAN ABSOLUTE ERROR	WITHIN 95% CONFIDENCE INTERVAL	
PACIFIC (6 SW)	.178 (Class 1)	.182	.009	83%	Single Moment Model Spill Control 250 Erlangs
	.593 (Class 2)	.600	.019	63%	
CONUS (10 SW)	.042 (Class 1)	.034	.025	32%	Single Moment Model Originating Office Control 400 Erlangs
	.611 (Class 2)	.642	.065	55%	
EUROPE (11 SW)	.050 (Class 1)	.047	.006	82%	Single Moment Model (with adjustment for peakedness) Originating Office Control
	.490 (Class 2)	.516	.036	61%	

Figure 5. Performance DCEC Voice Network Model with Preemption - Friendly Discipline

confidence interval was over three fourths (82%), a considerable improvement over the CONUS configuration (32%). This improvement was accomplished by incorporating a 5% increase of the non-terminating tandem class 1 traffic carried on each interswitch trunk in the model in order to account for peakedness of any overflow traffic.

Figure 6 shows a comparison of the central processing unit (CPU) time of an IBM 370/155 computer used during simulation and computer runs of the Voice Network Model with Preemption.

AREA	SIMULATION	ANALYTICAL MODEL
Pacific (6 Switches)	23 min.	8 sec.
CONUS (10 Switches)	35 min.	37 sec.

Figure 6. Comparison of IBM 370/155 CPU Time Used by Event-by-Event Simulator and the Voice Network Model with Preemption

In order to minimize computer turn-around time, five short runs were made for each simulation. A total of 216 thousand calls were processed for the Pacific and 338 thousand for the CONUS. A new random seed was selected for each run in order to eliminate any possible bias in the generation of traffic.

The CPU time to determine the performance of the full CONUS AUTOVON configuration (60 switches) using the unmodified Voice Network Model for a single class of traffic (5000 erlangs of ROUTINE traffic) is about 10 to 12 minutes. Since the Voice Network Model with Preemption may be viewed as two unmodified models running in parallel, one for class 1 traffic and the other for class 2 traffic, the CPU time for the Voice Network Model with Preemption for the full CONUS AUTOVON configuration should be no more than twice that of the unmodified model or 20 to 24 minutes. If the precedence traffic is much less than that of the ROUTINE traffic then the load assignment procedure for the precedence traffic will take considerably less time. There are about 200 to 300 erlangs of precedence traffic in the CONUS AUTOVON during the busy hour. Thus it may be estimated that the Voice Network Model with Preemption would use about 18 CPU minutes to estimate performance of the full CONUS AUTOVON configuration during the busy hour (12 minutes for ROUTINE traffic and 6 minutes for precedence traffic). Alternatively, it would not be unreasonable to estimate that the corresponding simulation would require about three CPU hours. Hence, one can see the substantial savings attainable with this analytic model.

## V. CONCLUSION AND FUTURE WORK

A Voice Network Model with Preemption has been developed by DCEC which will be used to perform future engineering studies of the AUTOVON involving preemption. These studies could only have been performed previously by lengthy computer simulation. Presently, the model will be used to assess the potential performance and economic impact of facsimile-imagery service upon the present and future AUTOVON. It will also be used to reduce the time required for performing FNB studies. All AUTOVON interswitch trunking, which must meet the FNB criteria, will initially be resized using the model in place of event-by-event computer simulation.

In order to increase the usefulness of the Voice Network Model with Preemption, the following extensions to the model are planned:

- (1) Extend the capability of the model to include both ruthless and friendly preemption disciplines in the same network.
- (2) Investigate and extend the model to include different holding times for each class of traffic.
- (3) Examine the precision of the model for networks similar to the ones generated in the FNB study.
- (4) Extend the capability of the model to include link reliability.
- (5) Develop a network design program for two classes of traffic which would complement the Voice Network Model with Preemption.

Both ruthless and friendly preemption disciplines exist in today's European AUTOVON; friendly preemption exists intra-Europe and ruthless preemption exists at the gateway switches between Europe and CONUS. Extending the capability of the model to include both ruthless and friendly preemption disciplines in the same network model will enable DCEC to more accurately assess the performance of the European AUTOVON.

In general, the holding times of each class of traffic may not have equal distributions as assumed. Thus, the effect of unequal holding times should be ascertained and incorporated in the model.

Based upon analyses performed to date, the precision of the Voice Network Model with Preemption is adequate for estimating network grades of service of not less than one or two blocked calls per 100 calls offered. One of the FNB criteria for AUTOVON is that no more than one FLASH call per 1000 FLASH calls offered can be blocked. Thus, in order to determine whether a network meets this criteria using the Voice Network Model with Preemption, the precision of the model will be studied and extended, if possible.



Appendix A.2, Link Performance for Unreliable Channels, outlines the theory necessary to extend the capability of the model to include link reliability. A link reliability of 99% is used in the computer FNB simulation runs. The Voice Network Model with Preemption, with the link reliability feature, may be useful in performing FNB studies as well as other network studies involving network reliability.

Finally, we would like to incorporate the model into the overall design and analysis algorithm currently being used at DCEC. This will extend DCEC's capability to design as well as to analyze communication networks capable of preemption.

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## APPENDIX

There are three subsections in this appendix. The first deals with the mathematical analysis and development of the link performance approximation for the friendly preemption discipline. In the second, a method for computing link performance when some of the channels in the link are not completely reliable is discussed. Finally, an example of the load reduction process used in the algorithm is given in the third subsection.

### A.1 LINK PERFORMANCE OF FRIENDLY PREEMPTION DISCIPLINE

Under this discipline, an arriving high precedence (class 1) call that is blocked on a link first tries to find an alternative path to the node at the other end of the link. If one exists, the call takes that path; if not, the call returns to the link and preempts a randomly selected lower precedence (class 2) call. If no class 2 calls are present, the class 1 call is alternately routed.

It is assumed there are  $s$  channels in the link; calls from each of the two classes arrive at the link in accordance with two independent Poisson processes with parameters  $\lambda_1$  and  $\lambda_2$ , respectively. The holding time for each class of calls is assumed to be the same and exponentially distributed with mean  $\mu$ . Arriving class 1 calls finding all the channels busy leave the link and try to find another path in the network. We assume there is a probability,  $\beta$ , that they will return. If the returning class 1 call finds all the channels occupied by class 1 calls, it is lost to the link. No queueing is allowed for either class. In the steady state condition, we are interested in determining the loss probabilities for class 1 and 2 calls.

In steady state, let  $Q_i$ ,  $i=1,2$ , be the random variable representing the number of class  $i$  calls on the channels, and  $Q=Q_1+Q_2$  the total number of calls on the channels. If  $\rho_i=\lambda_i/\mu$ ,  $i=1,2$ , and  $P_{i,j}=\Pr\{Q_1=i, Q_2=j\}$ , then the steady state equations for  $P_{i,j}$  are for  $i=0,1,\dots,s-1$  and  $j=0,1,2,\dots,s-1-i$ :

$$(\rho_1+\rho_2+i+j)P_{i,j}=\rho_1P_{i-1,j}+(i+1)P_{i+1,j}+(j+1)P_{i,j+1}+\rho_2P_{i,j-1}, \quad (A.1)$$

$$(\rho_1\beta+s)P_{i,s-i}=\rho_1P_{i-1,s-i}+\rho_2P_{i,s-i-1}+\rho_1\beta P_{i-1,s-i+1}; \quad (A.2)$$

and

$$sP_{s,0}=\rho_1P_{s-1,0}+\rho_1\beta P_{s-1,1}, \quad (A.3)$$



where

$$P_{-1,j} = P_{i,-1} = P_{s+1,j} = P_{i,s+1} = 0.$$

For the development of birth and death equations, see [4].

For  $k=0,1,\dots,s$ , let  $R_k = \Pr\{Q=k\}$ ; then from equation (A.1), it can be seen that  $R_k$  must satisfy the following set of equations:

$$(\rho_1 + \rho_2 + k)R_k = (\rho_1 + \rho_2)R_{k-1} + (k+1)R_{k+1}, \quad (A.4)$$

where  $R_{-1} = R_{s+1} = 0$ . The solution to these equations is, for  $k=0,1,\dots,s$

$$R_k = \Pr\{Q=k\} = \frac{\frac{(\rho_1 + \rho_2)^k}{k!}}{\sum_{j=0}^s \frac{(\rho_1 + \rho_2)^j}{j!}}. \quad (A.5)$$

From equation (A.5), it can be seen that the steady state probability that the number of calls on the channel is  $k$  does not depend on  $\beta$ , and hence, the total carried load, CL, is also invariant under  $\beta$  and is given by

$$CL = \sum_{k=0}^s kR_k = (\rho_1 + \rho_2)(1 - R_s). \quad (A.6)$$

By rewriting equation (A.1), one sees that the solution of equations (A.1)-(A.3) depends on  $s+1$  unknowns  $P_{0,0}, P_{0,1}, \dots, P_{0,s}$ . From  $R_0 = P_{0,0}$  and equation (A.3) the solution can be further reduced to the  $s-1$  unknowns  $P_{0,1}, P_{0,2}, \dots, P_{0,s-1}$ . One may then use equation (A.2) to obtain  $s-1$  equations in these unknowns. These equations were solved directly by an iterative method in order to develop an approximation for the probability of lost class 1 calls.

For each class of calls, it is desired to determine the probability that a call from that class will be lost on the link. The loss probability,  $PL_1$ , for class 1 calls is

$$PL_1 = (1-\beta)\Pr\{Q=s\} + \beta\Pr\{Q_1=s\}. \quad (A.7)$$

where, again,

$$Q = Q_1 + Q_2.$$

The first part on the right hand side of this equation represents the situation where the class 1 call is blocked on its friendly attempt to use the channels and does not return to preempt. The second part represents the preemption situation. For this case, class 1 calls are lost on the link when they return and find all the channels occupied by class 1 calls.

For class 2, the loss probability,  $PL_2$ , is given by

$$PL_2 = Pr\{Q=s\} + \frac{\rho_1 \beta [Pr\{Q=s\} - Pr\{Q_1=s\}]}{\rho_2} \quad (A.8)$$

The first factor in equation (A.8) represents the situation where all the channels are busy. The second is the proportion of class 2 calls that are preempted. In this expression, the numerator is the number of erlangs of class 1 calls that preempt the class 2 calls. This number must equal the number of class 2 calls that are preempted. Therefore, the proportion of class 2 calls preempted may be found by dividing this amount by the amount of class 2 offered traffic,  $\rho_2$ ; thus the second factor in equation (A.8) is determined. In equations (A.7) and (A.8) the probability,  $Pr\{Q=s\}$ , is given by equation (A.5) with  $k=s$ ; i.e.,  $R_s$ . The other probability,  $Pr\{Q_1=s\}$ , is given by  $P_{s,0}$ .

Several special cases are of interest. First, the situation where  $\beta=0$ ; i.e., class 1 and 2 calls equally contend for use of the channels. For this case, one can show (see [4]) that

$$P_{s,0} = \frac{\frac{\rho_1^s}{s!}}{\sum_{j=0}^s (\rho_1 + \rho_2)^j / j!} \quad (A.9)$$

The second special case is where  $\beta=1$ . This situation is equivalent to the protocol where an arriving class 1 call, which finds all channels busy, immediately preempts a class 2 call using a channel. This protocol has been analyzed in ref. [7]. One finds that

$$P_{s,0} = \frac{\frac{\rho_1^s}{s!}}{\sum_{j=0}^s \rho_1^j / j!} \quad (A.10)$$

These two special cases serve as bounds on  $P_{s,0}$ , and hence on  $PL_1$  and  $PL_2$ .

Another special case is where  $s=1$ . From equation (A.5), one can derive

$$P_{0,0} = \frac{1}{1+\rho_1+\rho_2},$$

from equation (A.3)

$$P_{0,1} = \frac{\rho_2}{(1+\rho_1\beta)(1+\rho_1+\rho_2)},$$

and finally, using equation (A.5),

$$P_{1,0} = \frac{\rho_1+\rho_1\beta(\rho_1+\rho_2)}{(1+\rho_1+\rho_2)(1+\rho_1\beta)}.$$

From these results, the loss probabilities may be found:

$$PL_1 = \frac{\rho_1(1+\beta(\rho_1+\rho_2))+\rho_2(1-\beta)}{(1+\rho_1+\rho_2)(1+\rho_1\beta)},$$

and

$$PL_2 = \frac{\rho_1+\rho_2+\rho_1\beta(1+\rho_1+\rho_2)}{(1+\rho_1+\rho_2)(1+\rho_1\beta)}.$$

The mathematical analysis presented so far requires one to solve a set of linear equations to determine  $P_{s,0}$ . The use of a model which would solve these linear equations would result in excessive computer run times. A simple approximation has been developed which eliminates the need to solve the set of linear equations and in turn significantly reduces the computer run time.

From equation (A.7) and (A.8), one sees that the only real problem in computing  $PL_1$  and  $PL_2$  is in determining  $\Pr\{Q_1=s\}=P_{s,0}$ . Thus, one needs to approximate this probability in order to implement these results in the network algorithm. From the numerical cases that were examined, with fixed values of  $\rho_1$ ,  $\rho_2$ , and  $s$ ,  $P_{s,0}$  appears to be a quadratic function in  $\beta$ ; thus,  $P_{s,0}$  can be approximated by the function  $f(\rho_1, \rho_2, s, \beta)$ , which is given by:

$$f(\rho_1, \rho_2, s, \beta) = C_0(\rho_1, \rho_2, s) + C_1(\rho_1, \rho_2, s)\beta + C_2(\rho_1, \rho_2, s)\beta^2, \quad (A.11)$$



where the constants  $C_i(\rho_1, \rho_2, s)$ ,  $i=0,1,2$ , are to be determined. Two equations in these unknowns can be immediately found. As was pointed out earlier, when  $\beta=0$

$$P_{s,0} = \frac{\rho_1^s / s!}{\sum_{j=0}^s (\rho_1 + \rho_2)^j / j!},$$

and when  $\beta=1$

$$P_{s,0} = \frac{\rho_1^s / s!}{\sum_{j=0}^s \rho_1^j / j!}.$$

From equation (A.11) this implies that

$$f(\rho_1, \rho_2, s, \beta) = C_0(\rho_1, \rho_2, s) = \frac{\rho_1^s / s!}{\sum_{j=0}^s (\rho_1 + \rho_2)^j / j!}, \quad (A.12)$$

and similarly

$$C_0(\rho_1, \rho_2, s) + C_1(\rho_1, \rho_2, s) + C_2(\rho_1, \rho_2, s) = \frac{\rho_1^s / s!}{\sum_{j=0}^s \rho_1^j / j!}. \quad (A.13)$$

One more equation is still required for determination of the constants  $C_i(\rho_1, \rho_2, s)$   $i=0,1,2$ . Examination of the numerical cases showed that at  $\beta=.6$ ,

$$f(\rho_1, \rho_2, s, .6) \sim \frac{\rho_1}{\rho_1 + \rho_2} f(\rho_1, \rho_2, s, 1) \quad (A.14)$$

where  $\sim$  signifies approximately. Using equations (A.11) and (A.14) one finds:

$$\rho_1 f(\rho_1, \rho_2, s, 1) = (\rho_1 + \rho_2) [C_0(\rho_1, \rho_2, s) + .6C_1(\rho_1, \rho_2, s) + .36C_2(\rho_1, \rho_2, s)].$$

The solution for  $C_j(\rho_1, \rho_2, s)$  is straightforward and is given by:

$$C_0(\rho_1, \rho_2, s) = \frac{\rho_1^s / s!}{\sum_{j=0}^s (\rho_1 + \rho_2)^j / j!}, \quad (A.15)$$

$$C_2(\rho_1, \rho_2, s) = \frac{5}{3} C_0(\rho_1, \rho_2, s) + \frac{.3\rho_2 - .2\rho_1}{.12(\rho_1 + \rho_2)} E(\rho_1, s), \quad (A.16)$$

and

$$C_1(\rho_1, \rho_2, s) = E(\rho_1, s) - C_0(\rho_1, \rho_2, s) - C_2(\rho_1, \rho_2, s), \quad (A.17)$$

where  $E(\rho_1, s)$  is Erlangs Loss Formula; i.e., the right hand side of equation (A.13). For the cases where  $\beta=0$  and  $\beta=1$ , one sees that the approximation is exact.

In tables A.1 through A.6 some numerical comparisons of the exact results to the approximations are given. For each of three cases,  $s=10, 15$  and  $20$ , the effect that different values of  $\beta$  have on light and heavy loadings were examined. For a fixed number of channels and fixed total offered load, three cases were considered. The first case assumed that the class 1 load was one-third of the class 2 load, the second assumed that the two loads were equal, and the last that the class 1 load was three times the class 2 load. For each of these cases and different values of  $\beta$ , comparisons of the exact (E) and approximate (A) results are given for  $P_{s,0}$ ,  $PL_1$ , and  $PL_2$ .

For all the cases presented in the tables, one sees that the approximation estimates  $PL_1$  and  $PL_2$  very well. When it was low for one of the loss probabilities, it was high for the other. Of course, this was to be expected since the total carried load is constant and the approximation is consistent in this regard. Another comment worthy of mention is the great effect  $\beta$  has on  $PL_1$  and  $PL_2$ . For example, consider Table A.2 with  $\rho_1=3$  and  $\rho_2=9$ ; when  $\beta=.2$   $PL_1=.2415$ , but  $PL_1=.0607$  with  $\beta=.8$ . It can be seen that the loss probability is reduced by a factor of four when  $\beta$  is increased by a factor of four.

These tables certainly demonstrate the effect that this preemption discipline has on the performance of the link. It tends to let more class 2 calls use the link and forces the class 1 calls to find other

TABLE A.1. LOW LOAD ( $\rho_1 + \rho_2 = 6$ ) FOR  $s=10$

$\rho_1 = 1.5, \rho_2 = 4.5$

$\beta$		$P_{s,0}$	$PL_1$	$PL_2$
.2	E	.0000	.0345	.0460
	A	.0000	.0345	.0460
.4	E	.0000	.0259	.0489
	A	.0000	.0259	.0489
.6	E	.0000	.0173	.0518
	A	.0000	.0173	.0518
.8	E	.0000	.0086	.0546
	A	.0000	.0086	.0546

$\rho_1 = 3, \rho_2 = 3$

$\beta$		$P_{s,0}$	$PL_1$	$PL_2$
.2	E	.0001	.0345	.0517
	A	.0001	.0345	.0517
.4	E	.0002	.0259	.0603
	A	.0002	.0259	.0603
.6	E	.0004	.0175	.0688
	A	.0004	.0175	.0688
.8	E	.0006	.0091	.0772
	A	.0006	.0091	.0772

$\rho_1 = 4.5, \rho_2 = 1.5$

$\beta$		$P_{s,0}$	$PL_1$	$PL_2$
.2	E	.0025	.0350	.0675
	A	.0044	.0354	.0663
.4	E	.0055	.0281	.1157
	A	.0063	.0282	.1062
.6	E	.0072	.0216	.1047
	A	.0079	.0219	.1060
.8	E	.0090	.0157	.1252
	A	.0092	.0161	.1244



TABLE A.2. HIGH LOAD ( $\rho_1 + \rho_2 = 12$ ) FOR  $s=10$

$\rho_1=3, \rho_2=9$

$\beta$		$P_{s,0}$	$PL_1$	$PL_2$
.2	E	.0000	.2415	.3221
	A	.0000	.2415	.3221
.4	E	.0000	.1811	.3422
	A	.0000	.1811	.3422
.6	E	.0001	.1208	.3623
	A	.0002	.1208	.3623
.8	E	.0002	.0605	.3824
	A	.0004	.0607	.3820

$\rho_1=6, \rho_2=6$

$\beta$		$P_{s,0}$	$PL_1$	$PL_2$
.2	E	.0024	.2420	.3618
	A	.0059	.2427	.3611
.4	E	.0079	.1837	.4183
	A	.0130	.1863	.4174
.6	E	.0172	.1311	.4728
	A	.0216	.1337	.4701
.8	E	.0294	.0839	.5199
	A	.0316	.0856	.5183

$\rho_1=9, \rho_2=3$

$\beta$		$P_{s,0}$	$PL_1$	$PL_2$
.2	E	.0543	.2524	.4505
	A	.0594	.2534	.4474
.4	E	.0925	.2181	.5532
	A	.0957	.2195	.5493
.6	E	.1240	.1951	.6223
	A	.1260	.1964	.6286
.8	E	.1486	.1793	.6698
	A	.1500	.1804	.6698

TABLE A.3. LOW LOAD ( $\rho_1 + \rho_2 = 12$ ) FOR  $s=15$  $\rho_1=3, \rho_2=9$ 

$\beta$		$P_{s,0}$	$PL_1$	$PL_2$
.2	E	.0000	.0686	.0914
	A	.0000	.0686	.0914
.4	E	.0000	.0514	.0972
	A	.0000	.0514	.0972
.6	E	.0000	.3430	.1028
	A	.0000	.3430	.1028
.8	E	.0000	.0172	.1086
	A	.0000	.0172	.1086

 $\rho_1=6, \rho_2=6$ 

$\beta$		$P_{s,0}$	$PL_1$	$PL_2$
.2	E	.0000	.0686	.1028
	A	.0001	.0686	.1028
.4	E	.0001	.0515	.1199
	A	.0002	.0514	.1199
.6	E	.0002	.0345	.1375
	A	.0004	.0345	.1369
.8	E	.0005	.0175	.1539
	A	.0007	.0177	.1538

 $\rho_1=9, \rho_2=3$ 

$\beta$		$P_{s,0}$	$PL_1$	$PL_2$
.2	E	.0028	.0691	.1355
	A	.0066	.0699	.1332
.4	E	.0070	.0544	.1808
	A	.0111	.0559	.1752
.6	E	.0112	.0410	.2120
	A	.0149	.0432	.2132
.8	E	.0155	.0296	.2542
	A	.0170	.0313	.2487

TABLE A.4. HIGH LOAD ( $\rho_1 + \rho_2 = 24$ ) FOR  $s=15$  $\rho_1=6, \rho_2=18$ 

B		$P_{s,0}$	$PL_1$	$PL_2$
.2	E	.0000	.3389	.4519
	A	.0000	.3389	.4519
.4	E	.0000	.2542	.4802
	A	.0000	.2542	.4802
.6	E	.0001	.1695	.5084
	A	.0002	.1696	.5084
.8	E	.0002	.0849	.5366
	A	.0005	.0851	.5365

 $\rho_1=12, \rho_2=12$ 

B		$P_{s,0}$	$PL_1$	$PL_2$
.2	E	.0012	.3392	.5082
	A	.0011	.3412	.5061
.4	E	.0091	.2578	.5895
	A	.0257	.2645	.5829
.6	E	.0283	.1864	.6609
	A	.0486	.1952	.6522
.8	E	.0556	.1292	.7181
	A	.0628	.1350	.7123

 $\rho_1=18, \rho_2=6$ 

B		$P_{s,0}$	$PL_1$	$PL_2$
.2	E	.0696	.3529	.6362
	A	.0850	.3560	.6267
.4	E	.1483	.3135	.7542
	A	.1519	.3149	.7498
.6	E	.2060	.2931	.8155
	A	.2056	.2928	.8163
.8	E	.2459	.2814	.8504
	A	.2463	.2817	.8493



TABLE A.5. LOW LOAD ( $\rho_1 + \rho_2 = 18$ ) FOR  $s=20$ 

$$\rho_1 = 4.5, \rho_2 = 13.5$$

$\beta$		$P_{s,0}$	$PL_1$	$PL_2$
.2	E	.0000	.0874	.1165
	A	.0000	.0874	.1165
.4	E	.0000	.0655	.1238
	A	.0000	.0655	.1238
.6	E	.0000	.0437	.1311
	A	.0000	.0437	.1311
.8	E	.0000	.0219	.1383
	A	.0000	.0219	.1383

$$\rho_1 = 9, \rho_2 = 9$$

$\beta$		$P_{s,0}$	$PL_1$	$PL_2$
.2	E	.0000	.0874	.1311
	A	.0000	.0874	.1311
.4	E	.0000	.0655	.1529
	A	.0001	.0656	.1528
.6	E	.0001	.0437	.1747
	A	.0003	.0438	.1745
.8	E	.0003	.0221	.1964
	A	.0004	.0222	.1962

$$\rho_1 = 13.5, \rho_2 = 4.5$$

$\beta$		$P_{s,0}$	$PL_1$	$PL_2$
.2	E	.0021	.0876	.1708
	A	.0073	.0888	.1704
.4	E	.0060	.0678	.2326
	A	.0130	.0708	.2246
.6	E	.0113	.0505	.2854
	A	.0177	.0543	.2739
.8	E	.0174	.0358	.3295
	A	.0212	.0388	.3204

TABLE A.6. HIGH LOAD ( $\rho_1 + \rho_2 = 36$ ) FOR  $s=20$

$\rho_1=9, \rho_2=27$

$\beta$		$P_{s,0}$	$PL_1$	$PL_2$
.2	E	.0000	.3781	.5041
	A	.0000	.3781	.5041
.4	E	.0000	.2836	.5356
	A	.0000	.2836	.5356
.6	E	.0000	.1891	.5672
	A	.0001	.1891	.5671
.8	E	.0001	.0946	.5986
	A	.0003	.0948	.5985

$\rho_1=18, \rho_2=18$

$\beta$		$P_{s,0}$	$PL_1$	$PL_2$
.2	E	.0004	.3782	.5671
	A	.0145	.3810	.5642
.4	E	.0077	.2867	.6585
	A	.0327	.2967	.6486
.6	E	.0319	.2082	.7317
	A	.0546	.2218	.7234
.8	E	.0690	.1498	.7955
	A	.0801	.1586	.7867

$\rho_1=27, \rho_2=9$

$\beta$		$P_{s,0}$	$PL_1$	$PL_2$
.2	E	.0712	.3923	.7135
	A	.0970	.3976	.6976
.4	E	.1749	.3535	.8299
	A	.1778	.3547	.8264
.6	E	.2459	.3366	.8808
	A	.2421	.3343	.8876
.8	E	.2917	.3279	.9068
	A	.2903	.3268	.9100

paths in the network. This fact is brought out in Table A.2 for the case where  $\rho_1=9$  and  $\rho_2=3$ . In the ruthless discipline the class 1 calls are allowed to preempt the class 2 calls immediately; the loss probability for class 2 would be greater than .7. But under the friendly discipline, if  $\beta=.2$ , then  $PL_2=.44$ . Thus more class 2 calls are able to use the link.

## A.2 LINK PERFORMANCE FOR UNRELIABLE CHANNELS

As part of the Flash Non-Blocking Study, the capability is required to determine the link performance in terms of loss probability when the channels on the link fail. Currently, this situation is handled for the Flash Non-Blocking Study by simulating this process in a Monte Carlo manner. That is, suppose there are  $s$  channels on the link and for the time period under consideration, let  $p$  be the probability that a channel on the link is working. For the Flash Non-Blocking Study and for each channel in a Monte Carlo run, a uniform random number is drawn and compared with  $p$  in order to determine if the channel is available. Then, the number of channels available on each link is used in an event-by-event simulation of the system. After the simulation is in a steady state, statistics are taken and the process is repeated. That is,  $s$  more uniformly distributed random numbers are drawn and compared with the  $p$ 's. This method is continued until some statistical confidence is reached.

In this appendix, it is shown how to solve the problem mathematically. Again let  $s$  be the number of channels and  $\rho$  the offered load in erlangs, generated by a Poisson process. Assume there is only one class of customers for this comparison. If  $p$  is the probability a channel is available over the time interval of interest, then the probability that there are  $n$  channels available is given by a binomial distribution. If there are  $n$  channels available, then Erlang's Loss Formula  $E(\rho, n)$  is the equation for the loss probability. Using conditional probability statements one arrives at the loss probability,  $PL$ , for the system as:

$$PL = \sum_{n=0}^s \binom{s}{n} p^n (1-p)^{s-n} E(\rho, n). \quad (A.18)$$

Table A.7 gives a comparison of the use of equation (A.18) and the simulation method discussed earlier in the subsection. The parameter  $NK$  represents the number of Monte Carlo runs required to achieve the level of accuracy shown in the table. As one can see, equation (A.18) gives the required solution and does not necessitate the substantial run times of the simulation.

## A.3 LOAD REDUCTION PROCESS\*

A call that does not find a free path in a network may seize several links in tandem before finally being blocked. After blockage, the call

\*This process was developed by Mr. Dave Garbin, when he worked at DCEC.



TABLE A.7 LOSS PROBABILITY COMPARISONS FOR UNRELIABLE CHANNELS ( $s=20, \rho=10$ )

NK \ P	.8	.9	.95	.99
1000	.03113	.00969	.00511	.00222
2000	.03176	.00946	.00494	.00227
3000	.03150	.00963	.00483	.00230
4000	.03133	.00964	.00479	.00229
5000	.03100	.00963	.00472	.00228
6000	.03098	.00951	.00470	.00227
7000	.03082	.00952	.00466	.00228
8000	.03067	.00964	.00463	.00228
9000	.03047	.00966	.00465	.00229
10000	.03049	.00965	.00466	.00228
11000	.03048	.00965	.00466	.00228
12000	.03052	.00964	.00463	.00228
13000	.03037	.00961	.00460	.00227
14000	.03037	.00959	.00465	.00227
15000	.03053	.00958	.00465	.00227
16000	.03059	.00960	.00464	.00227
17000	.03058	.00961	.00464	.00227
18000	.03051	.00962	.00463	.00227
19000	.03050	.00958	.00463	.00227
20000	.03042	.00957	.00462	.00227
Eq. (A. 18)	.03037	.00963	.00460	.00226

will be dropped from the preceding links. Because it was assumed that the set up time is zero a blocked call will not appear as offered traffic to those links where it found a free channel, and the offered load to each link must be reduced to reflect this fact.

An example network is shown in Figure A.1. For now, assume that there is one class of calls in the network. The distribution over all links of traffic originating at the source node S destined for node D,  $\rho_{SD}$ , is desired. According to the routing table shown in Figure A.1, all traffic from node S passing through node 1 first attempts to proceed from node 1 directly to node D. If any traffic is blocked, it is next routed via node 2 on to node D. The traffic initially offered to each link may be expressed as:

$$\begin{aligned}\rho_{S1} &= \rho_{SD}, & \rho_{1D} &= \rho_{SD}(1 - PB_{S1}) \\ \rho_{12} &= \rho_{1D}PB_{1D}, \text{ and} & \rho_{2D} &= \rho_{12}(1 - PB_{12}),\end{aligned}\tag{A.19}$$

where  $PB_{XY}$  is the blocking probability on link XY.

The traffic reaching the destination node is

$$\rho_D = \rho_{1D}(1 - PB_{1D}) + \rho_{2D}(1 - PB_{2D}).\tag{A.20}$$

Assuming the following initial values of link blocking,

$$PB_{S1} = .05, \quad PB_{1D} = .1\tag{A.21}$$

$$PB_{12} = .15, \text{ and } PB_{2D} = .2$$

and that the offered traffic  $\rho_{SD} = 10$ , it follows from (A.19) and (A.20) that:

$$\rho_{S1} = 10.00, \quad \rho_{1D} = 9.50,\tag{A.22}$$

$$\rho_{12} = .95, \quad \rho_D = 9.20 \text{ and } \rho_{2D} = .81.$$

The traffic blocked at each node is given by:

$$\rho_S = \rho_{SD}PB_{S1} = .5\tag{A.23}$$

$$\rho_2 = \rho_{2D}PB_{2D} = .16$$

$$\rho_1 = \rho_{12}PB_{12} = .14$$

The traffic lost at node 2 (in going from 2 to D),  $\rho_2$ , had been carried by the link between nodes 1 and 2. Hence the initial estimate of the offered traffic to link 1,2 has to be reduced as follows;

$$\bar{\rho}_{12} = \rho_{12} - \frac{\rho_2}{1 - PB_{12}} \quad (A.24)$$

This is the original estimate of the offered load on link 1,2 minus the offered load corresponding to the traffic that was blocked on link 2,D.

Similarly, for the link between nodes S and 1, we have:

$$\bar{\rho}_{S1} = \rho_{S1} - \frac{\rho_1 + \rho_2}{1 - PB_{S1}} \quad (A.25)$$

Substituting values from above yields:

$$\begin{aligned} \bar{\rho}_{12} &= .76 \\ \bar{\rho}_{S1} &= 9.68. \end{aligned} \quad (A.26)$$

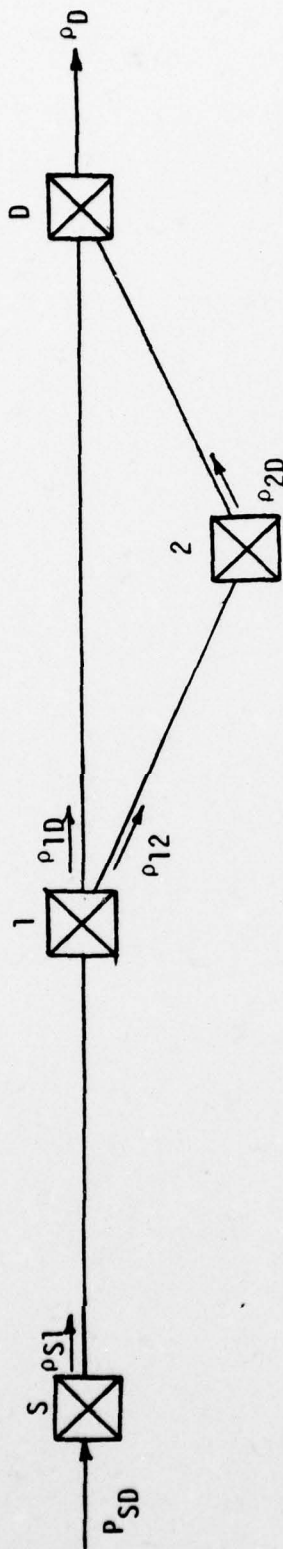
No load reduction is made to the traffic offered the link between nodes 1 and D as well as to the traffic offered the link between nodes 2 and D. Neither link carries any traffic which eventually is blocked within the network; thus,

$$\begin{aligned} \bar{\rho}_{1D} &= \rho_{1D} & \bar{\rho}_{2D} &= \rho_{2D} \\ & & \text{and} & \\ &= 9.50 & &= .81. \end{aligned} \quad (A.27)$$

This procedure is repeated until the traffic between all other source to destination nodes has been distributed (including load reduction), and accumulated for each link. Link blocking probabilities are recomputed using the new estimates of traffic offered each link at the end of the load assignment procedure.

For the case where one class of traffic can preempt another, the preempted traffic is load reduced as if it were a blocked traffic.





FROM SW		ROUTER		
		S	1	2
T0	1st Choice	1	D	D
	2nd Choice	0	2	0
SW				

Figure A.1. Example Network Showing Load Reduction Process

## DISTRIBUTION LIST

### STANDARD:

R100 - 2	R200 - 1
R102/R103/R103R - 1	R300 - 1
R102M - 1	R400 - 1
R102T - 9 (8 for stock)	R500 - 1
R104 - 1	R700 - 1
R110 - 1	R800 - 1
R123 - 1 (Library)	NCS-TS - 1
R124A - 1 (for Archives)	101A - 1

205 - 13

DCA-EUR - 2 (Defense Communications Agency European Area  
ATTN: Technical Director  
APO New York 09131)

DCA-PAC - 1 (Defense Communications Agency Pacific Area  
ATTN: Technical Director  
Wheeler AFB, HI 96854)

USDCFO - 1 (Chief, USDCFO/US NATO  
APO New York 09667)